Student Guide

How Long will the Sun Live?

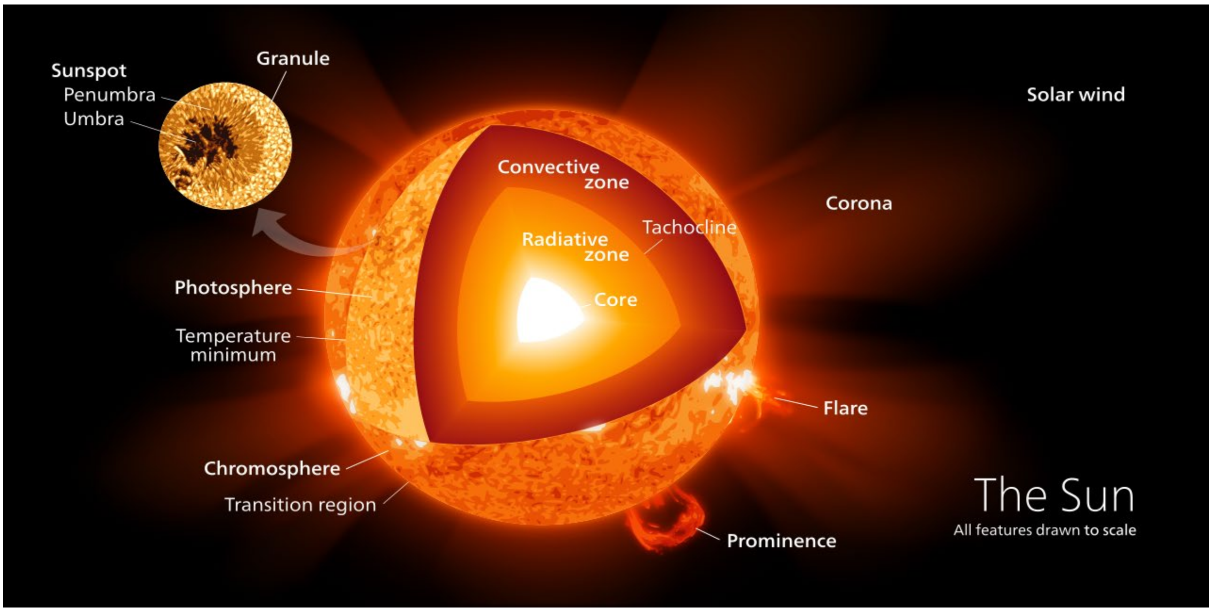
Calculating the Sun's Lifespan

Classroom Activity

Outline

In the nineteenth century scientists knew, from the rocks, that the Earth was millions of years old or more. Thus, the Sun had to have existed for the same time, but no one knew what kind of fuel powered the Sun for so long. In the twentieth century, scientists such as Hans Bethe and Arthur Eddington discovered that the Sun converts hydrogen to helium in its core, under enormous pressure and temperature. This stage in the Sun's life is called the Main Sequence. When the core hydrogen runs out the Sun will swell up into a giant and new fusion sequences will start. In this process, inner planets, like the Earth, will be damaged, even destroyed.

So, how long before this happens? We will carry out a series of calculations to find the answer. Throughout we will use SI units; kilograms, metres, seconds.



Procedure:

1 Finding the mass 'M_s' of the Sun.

To find 'M_s' we are going to use Kepler's Third Law, devised in the C17th and later modified by Newton. The speed of a planet orbiting the Sun and the time T to complete one orbit (year or period) depends on its distance 'a' (this is the semi-major axis of the orbit, equivalent to radius) and the Sun's mass 'M_s'.

To carry out this step we need the distance from Earth to the Sun. This was first estimated by Aristarchus in the 2nd century BC but the first accurate value was obtained during a transit of Venus in 1761. To work this out would take too long here. The distance 'a' is 1.496×10^{11}m and is also known as the Astronomical Unit (AU). We also need the Universal Gravitational Constant 'G', a measure of the intensity of the gravitational force throughout the universe; it is $6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^2$. Finally, we need the time 'T' for the Earth to complete one orbit, i.e. a year. In seconds, this is:

T = 365.25 days/year x 24 hours/day x 3600 seconds/ hour =

Using Newton's version of Kepler's Third Law:

$$T^2 = 4\pi^2a^3 / GM \quad \text{so} \quad M = 4\pi^2a^3 / GT^2$$

M_s =

In fact we have simplified the calculation by assuming the Earth's mass is tiny compared to the Sun, which it is. At the end of this exercise there is the option to investigate this in Addendum 1.

Fusion takes place in the Sun's core only. Helioseismology reveals this to be the central 10% of the Sun's entire mass. Place your core mass value below.

M_{core} = 0.1 x M_s =

2 Finding the power of the Sun.

The power of the Sun 'P' is the amount of energy it emits each second, measured in watts. The power is also known as the Sun's luminosity. The Sun's energy is sent outwards in all directions as an expanding invisible sphere of electromagnetic radiation. Only some hits the Earth, the rest goes in other directions.

A satellite, above the Earth's atmosphere can measure the how much of the Sun's power is absorbed per square metre of collecting surface. This is called the solar constant 'I'. The satellite, like the Earth itself, is one astronomical unit (1 AU), i.e. 1.496×10^{11}m from the source of energy, the Sun. The average value of the solar constant 'I' is known to be about 1360 watts per square metre (Wm^{-2}). Note the word 'average'; the Sun's output varies a little due to sunspots.

To find the power/luminosity going in all directions we need to extrapolate, from one square metre, the power over the entire surface of a theoretical sphere of radius 1 AU.

The area of a sphere of radius r = 1.496×10^{11}m (= 1AU) is:

$A = 4\pi r^2 =$

So, to get the total solar power we multiply our solar constant by this area:

$P = A \times I =$

Check your answers so far.

3 Finding the mass consumed by fusion in the Sun's core.

To produce the power, radiated into space the Sun has to convert mass 'm' into energy 'E'. For E we will use the power P from the previous calculation to get the mass used per second. Using 'c', the speed of light in a vacuum (2.997 x 10^8 ms^{-1}), Einstein's equation gives the relationship:

$$E = mc^2 \quad \text{so} \quad m = E / c^2 =$$

4 How much hydrogen is used in core fusion every second?

In step 3 we found the mass converted to energy, but this is only part of the mass of the hydrogen atoms used up every second in core fusion. The rest is converted to helium and positrons e^+ (positive versions of electrons). The main fusion process in the Sun's core (called ppI) has three steps but can be summarised in a single line:

4 hydrogen nucleus ➔ 1 helium nucleus + 2 positrons + solar energy

i.e. 4H ➔ 1 He + 2e$^+$ + E

We need to find out what proportion of the mass involved in this process has been converted to solar energy. That can be determined by two calculations; the first is a subtraction:

Mass of 4H – mass of 1 He – mass of 2e$^+$ = mass converted to solar energy

For convenience we will perform this calculation in small quantities, called atomic mass units (U). One U = 1.67 x 10^{-27}kg. Here are the masses for the components:

Hydrogen	**1.00784 U**
Helium	**4.00260 U**
Positron	**0.00059 U**

Using the data, find the mass of the solar energy released in atomic mass units, 'm_u'.

m_u =

Now divide this value m_u by the mass of the 4 hydrogen atoms used in the process to get the proportion p. You can also express this value as a percentage although we will not use that in further calculations. In a sense we are finding the efficiency of the reaction; only matter anti-matter annihilations are truly 100% efficient. This fusion reaction is much lower.

$p = m_u / (4 \times 1.00784) =$

p as a percentage = p x 100 = %

Now we will find the total mass of core hydrogen 'm_h' used every second in the fusion process. (This is, of course a much larger figure than the amount 'm' converted to solar power because much of it has become helium.)

Hydrogen mass m_h consumed per second =

mass converted to energy, m (from step3) / proportion p*(step 4)

Check your answers so far.

5 How long does that hydrogen last?

So, we now have the mass of hydrogen ‘M_{core}’ in the Sun’s core, in kg, and the rate at which it is used ‘m_h’ in kg s^{-1}. To find the total time for all the hydrogen to be consumed, i.e. the lifespan of the Sun:

Sun’s lifespan = M_{core} / m_h = seconds

This is an enormous time scale so we will convert it to years. You calculated how many seconds there are in one year in step 1. You can use that value here.

Sun’s lifespan (in years) =

6 How old is the Sun now and how much time is left?

The oldest rocks on Earth radiometrically date to over 4 billion years but even older primordial debris from the formation of the Sun and solar system still exists and falls to Earth as chondritic meteorites. Look up the age of chondritic meteorites on the internet (e.g. https://en.wikipedia.org/wiki/Chondrite) Other languages available by replacing ‘en’ with ‘fr’, ‘de’ etc.

Time remaining = Sun’s lifespan – age of chondritic meteorites =

7

Is this answer correct?

Assumptions have been made about the data used. Consider any factors that may affect your answer.

Further Activities:

Strictly speaking, Kepler's third law calculates the mass of both the Sun and the Earth but can we ignore the Earth because its mass is insignificant compared to the Sun?

To find the mass of the Earth we can weigh a 1 kilogram mass 'm' and find that its weight (a force) 'F' is 9.81N (this varies very slightly with altitude and latitude but that won't make a significant difference). The circumference, and thus the radius, of the Earth was measured by Eratosthenes and Al Biruni, around 240BC and 1000AD respectively. The Earth's radius 'r_e' is 6.371×10^6 metres. We also need the Universal Gravitational Constant 'G', a measure of the intensity of the gravitational force throughout the universe; it is 6.67×10^{-11} m^3 kg^{-1} s^{-2}.

Using Newton's gravitational equation we can find the mass M_e of the Earth:

$$F = GM_e m/r^2 \quad \text{so } M_e = Fr^2/Gm$$

M_e =

Compared to the Sun, the Earth's mass is:

Earth's mass M / Sun's mass M_s = solar masses

Comment on the significance of the Earth's mass.

The text mentions several scientists from history:

- Aristarchus
- Eratosthenes
- Al Biruni
- Johannes Kepler
- Isaac Newton
- Albert Einstein
- Arthur Eddington
- Hans Bethe

You could research any one of them and create a poster about their life and work.